

(Some of the) Particle Physics on the Way to the Muon Collider

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Outline

1. Steps Toward a TeV Muon Collider – Theorist's View (Very Brief);
2. Learning About/From Muons;
3. Learning About/From Neutrinos;
4. Concluding Thoughts.

The Muon Path to Energy Frontier is Intense

If we are ever to build a weak-scale muon collider, we will need to learn how to build, for a finite amount of money, . . .

. . . a multi MW proton source

. . . muon beams

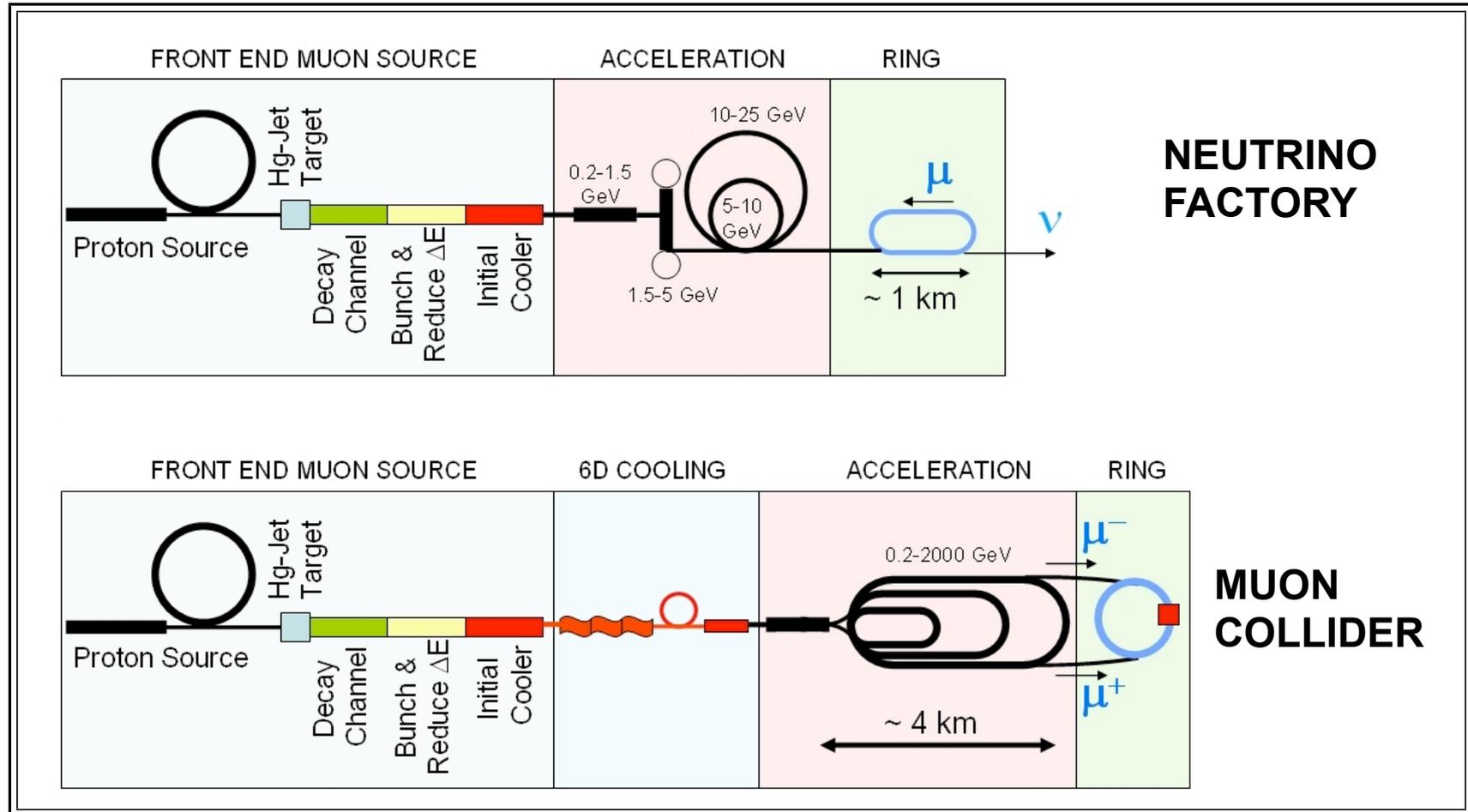
. . . muon storage rings

. . . etc.

The physics case for every one of these components is quite strong in its own right. [IMHO]



as we have been learning all week...

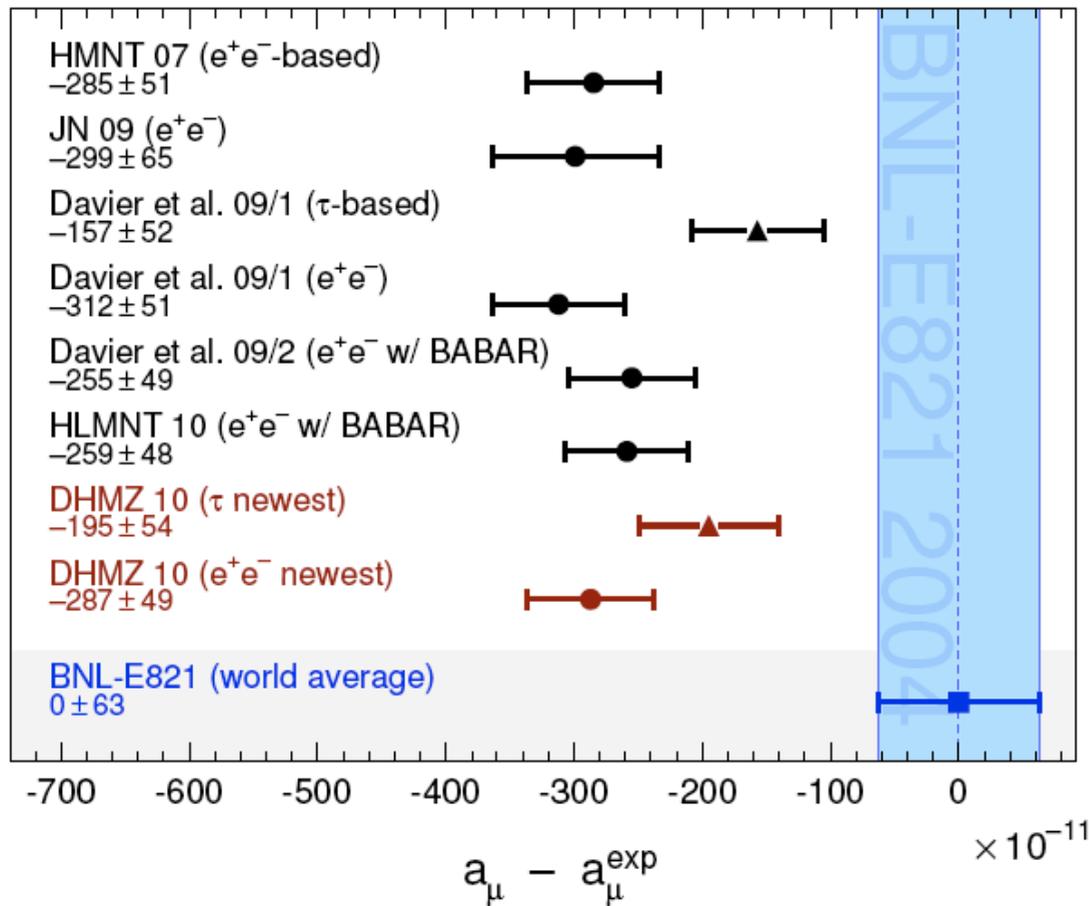


[S. Geer's talk this morning]

MUONS

Precision measurement of muon properties, and searches for very rare muon processes:

- The muon magnetic dipole moment.
- The muon electric dipole moment.
- Charged-lepton flavor violating muon processes: $\mu \rightarrow e$ -conversion in nuclei *et al.*



NOTE: $a_\mu^{LbL} = 105 \pm 26 \times 10^{-11}$

FIG. 9: Compilation of recent results for a_μ^{SM} (in units of 10^{-11}), subtracted by the central value of the experimental average [12, 57]. The shaded vertical band indicates the experimental error. The SM predictions are taken from: this work (DHMZ 10), HLMNT (unpublished) [58] (e^+e^- based, including BABAR and KLOE 2010 $\pi^+\pi^-$ data), Davier *et al.* 09/1 [15] (τ -based), Davier *et al.* 09/1 [15] (e^+e^- -based, not including BABAR $\pi^+\pi^-$ data), Davier *et al.* 09/2 [10] (e^+e^- -based including BABAR $\pi^+\pi^-$ data), HMNT 07 [59] and JN 09 [60] (not including BABAR $\pi^+\pi^-$ data).

[Davier *et al.*, 1010.4180]

Sensitivity to New Physics

If there is new ultra-violate physics, it will manifest itself, as far as a_μ is concerned, via the following effective operator (dimension 6):

$$\frac{\lambda H}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu} \rightarrow \frac{m_\mu}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu},$$

where Λ is a proxy for the new physics scale. (dependency on muon mass is characteristic of several (almost all?) models. It is NOT guaranteed)

Contribution to a_μ from operator above is

$$\delta a_\mu = \frac{4m_\mu^2}{e\Lambda^2}$$

Current experimental sensitivity: $\Lambda \sim 10$ TeV.

Note that, usually, the new physics scale can be much lower due to loop-factors, gauge couplings, etc. In the SM the heavy gauge boson contribution yields

$$\frac{1}{\Lambda^2} \sim \frac{eg^2}{16\pi^2 M_W^2} \longrightarrow \delta a_\mu \sim \frac{m_\mu^2 G_F}{4\pi^2} \quad \text{Not A Bad Estimate!}$$

On the muon electric-dipole moment, d_μ

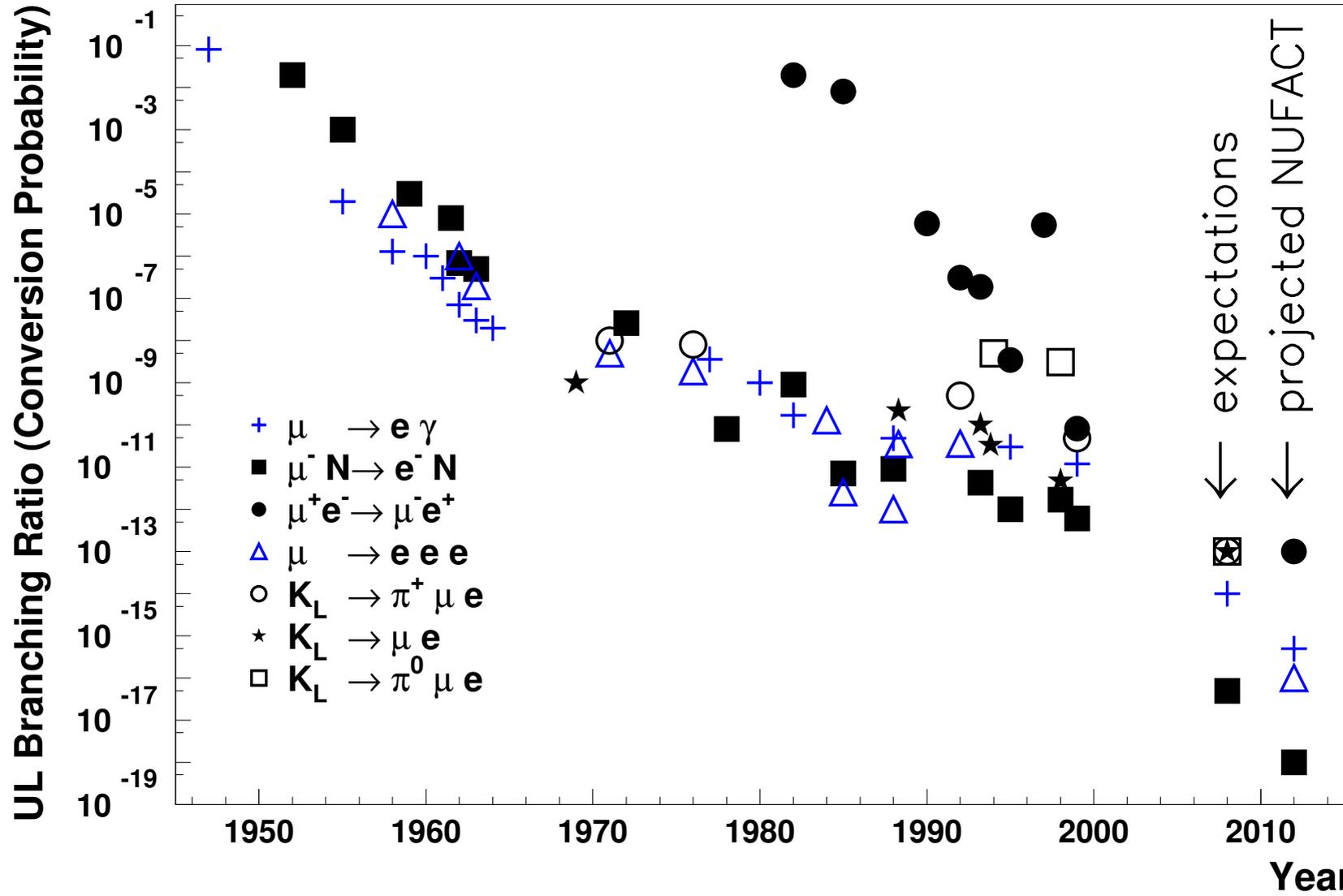
- CP-violating observable.
- Predicted to be non-zero-but-tiny in the SM: $d_\mu < 10^{-36}$ e-cm. Great place to look for new physics!
- Current bound: $d_\mu < 1.8 \times 10^{-19}$ e-cm. Compare to $d_e < 10^{-27}$ e-cm.
- In general, $d_\ell \propto m_\ell$, so $d_\mu \sim d_e \times (m_\mu/m_e)$.
- New $g - 2$ experiment at FNAL would be sensitive to $d_\mu > 10^{-21}$ e-cm. Dedicated effort could reach $d_\mu > 10^{-24}$ e-cm. Is it worth it?
- Same effective operator contributes to a_μ and d_μ

$$\frac{m_\mu}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu} \quad \text{versus} \quad \epsilon_{\text{CP}} \frac{m_\mu}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \gamma_5 \mu F^{\mu\nu}.$$

ϵ_{CP} measures how much the new physics violates CP.

If $\Lambda \sim 10$ TeV, $\epsilon_{\text{CP}} \ll 1$.

Searches for Lepton Number Violation (μ and e)



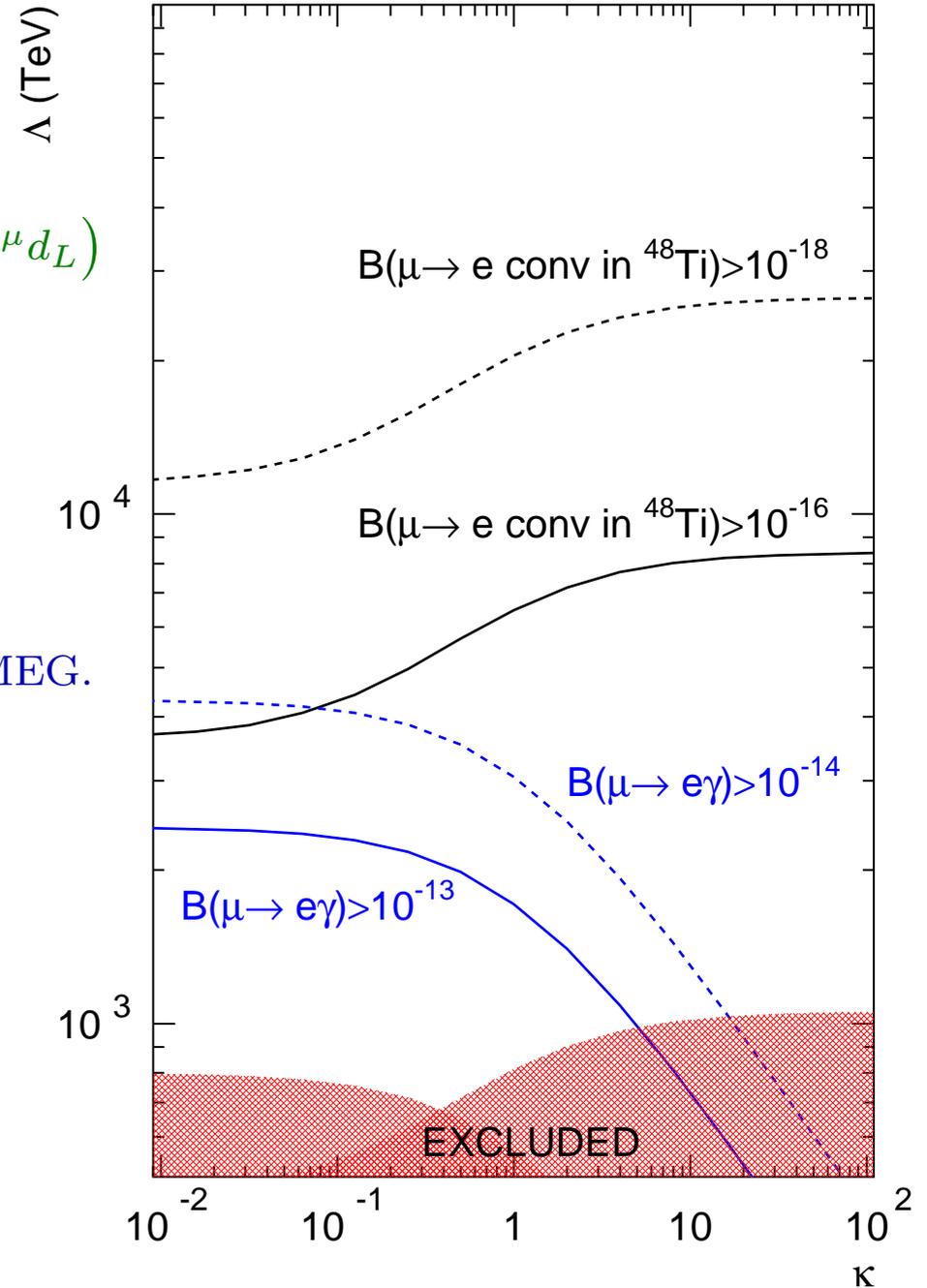
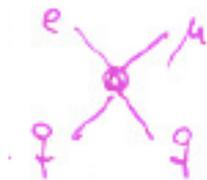
[hep-ph/0109217]

Model Independent Considerations

$$L_{\text{CLFV}} = \frac{m_\mu}{(\kappa+1)\Lambda^2} \bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu} + \frac{\kappa}{(1+\kappa)\Lambda^2} \bar{\mu}_L \gamma_\mu e_L (\bar{u}_L \gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L)$$

- $\mu \rightarrow e$ -conv at 10^{-17} “guaranteed” deeper probe than $\mu \rightarrow e\gamma$ at 10^{-14} .
- We don’t think we can do $\mu \rightarrow e\gamma$ better than 10^{-14} . $\mu \rightarrow e$ -conv “only” way forward after MEG.
- If the LHC does not discover new states $\mu \rightarrow e$ -conv among very few process that can access 1000+ TeV new physics scale:

tree-level new physics: $\kappa \gg 1, \frac{1}{\Lambda^2} \sim \frac{g^2 \theta_{e\mu}}{M_{\text{new}}^2}$.



Model Independent Comparison Between $g - 2$ and CLFV:

The dipole effective operators that mediate $\mu \rightarrow e\gamma$ and contribute to a_μ are virtually the same:

$$\frac{m_\mu}{\Lambda^2} \bar{\mu} \sigma^{\mu\nu} \mu F_{\mu\nu} \quad \times \quad \theta_{e\mu} \frac{m_\mu}{\Lambda^2} \bar{\mu} \sigma^{\mu\nu} e F_{\mu\nu}$$

$\theta_{e\mu}$ measures how much flavor is violated. $\theta_{e\mu} = 1$ in a flavor indifferent theory, $\theta_{e\mu} = 0$ in a theory where individual lepton flavor number is exactly conserved.

If $\theta_{e\mu} \sim 1$, $\mu \rightarrow e\gamma$ is a much more stringent probe of Λ .

On the other hand, if the current discrepancy in a_μ is due to new physics,

$$\theta_{e\mu} \ll 1 \quad (\theta_{e\mu} < 10^{-4}).$$

[Hisano, Tobe, hep-ph/0102315]

e.g., in SUSY models, $Br(\mu \rightarrow e\gamma) \simeq 3 \times 10^{-5} \left(\frac{10^{-9}}{\delta a_\mu} \right) \left(\frac{\Delta m_{\tilde{e}\tilde{\mu}}^2}{\tilde{m}^2} \right)^2$

Comparison restricted to dipole operator. If four-fermion operators are relevant, they will “only” enhance rate for CLFV with respect to expectations from $g - 2$.

What is This Good For?

While specific models provide estimates for the rates for CLFV processes, the observation of one specific CLFV process cannot determine the underlying physics mechanism (this is always true when all you measure is the coefficient of an effective operator).

Real strength lies in combinations of different measurements, including:

- kinematical observables (e.g. angular distributions in $\mu \rightarrow eee$);
- other CLFV channels;
- neutrino oscillations;
- measurements of $g - 2$ and EDMs;
- collider searches for new, heavy states;
- etc.

Regardless, a positive signal of CLFV may provide priceless guidance towards the next energy scale!

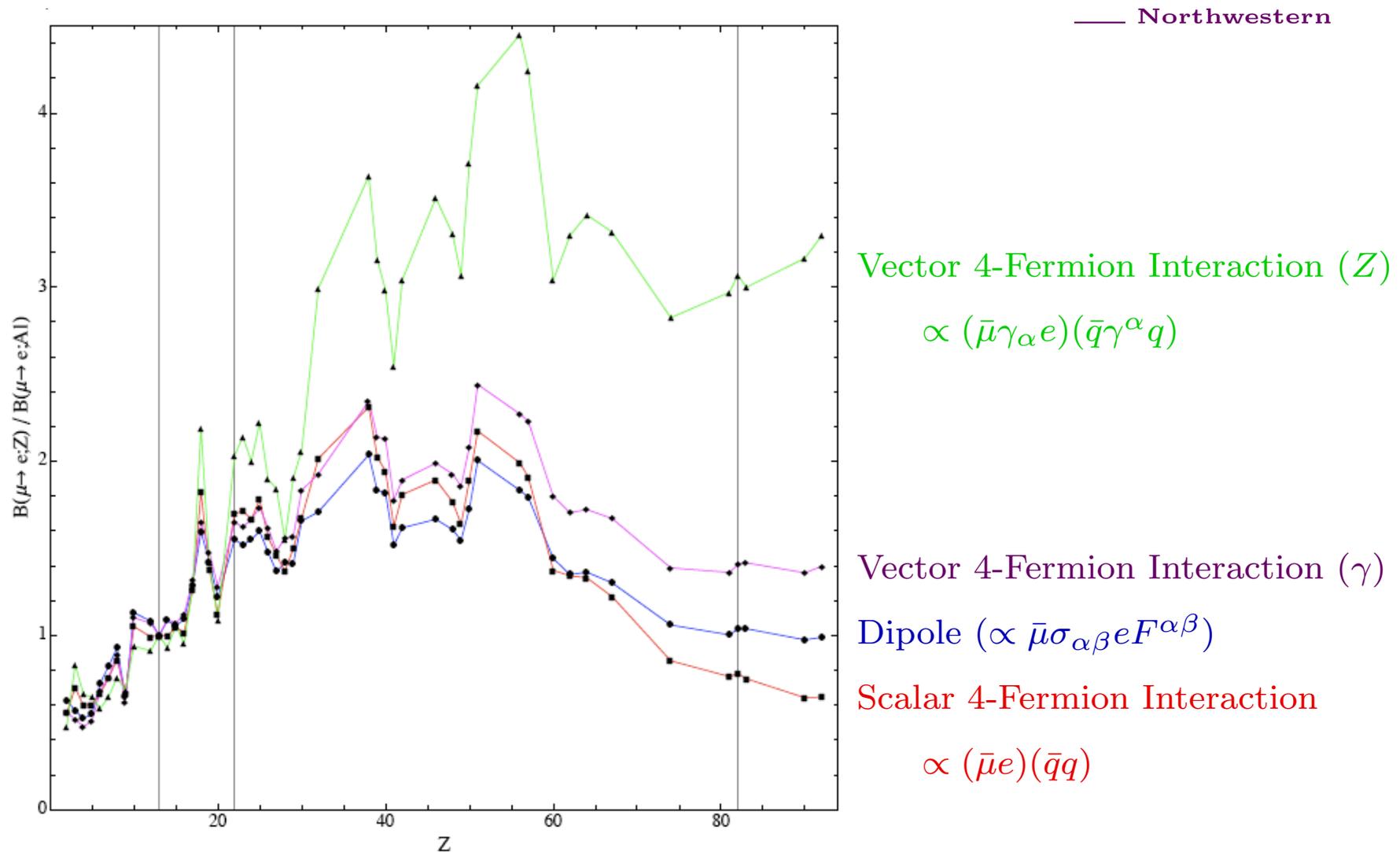


Figure 3: Target dependence of the $\mu \rightarrow e$ conversion rate in different single-operator dominance models. We plot the conversion rates normalized to the rate in Aluminum ($Z = 13$) versus the atomic number Z for the four theoretical models described in the text: D (blue), S (red), $V^{(\gamma)}$ (magenta), $V^{(Z)}$ (green). The vertical lines correspond to $Z = 13$ (Al), $Z = 22$ (Ti), and $Z = 83$ (Pb).

Along the way to the Muon Collider we expect ...

- Mu2e and COMET: $\mu \rightarrow e$ -conversion at 10^{-16} .
- $g - 2$ measurement a factor of 3–4 more precise.
- Project X-like: $\mu \rightarrow e$ -conversion at 10^{-18} (or precision studies?).
- Project X-like: deeper probe of muon edm.
- Muon Beams/Rings: $\mu \rightarrow e$ -conversion at 10^{-20} ? Revisit rare muon decays ($\mu \rightarrow e\gamma$, $\mu \rightarrow eee$) with new idea?

NEUTRINOS

Precision measurements of lepton mixing parameters, precision measurements of neutrino interactions, searches for other light neutral fermions, searches for the origin of neutrino masses, etc.

- The three-neutrinos paradigm.
- What is going on at short baselines?

Three Flavor Mixing Hypothesis Fits All* Data Really Well.

⇒ Good Measurements of Oscillation Observables

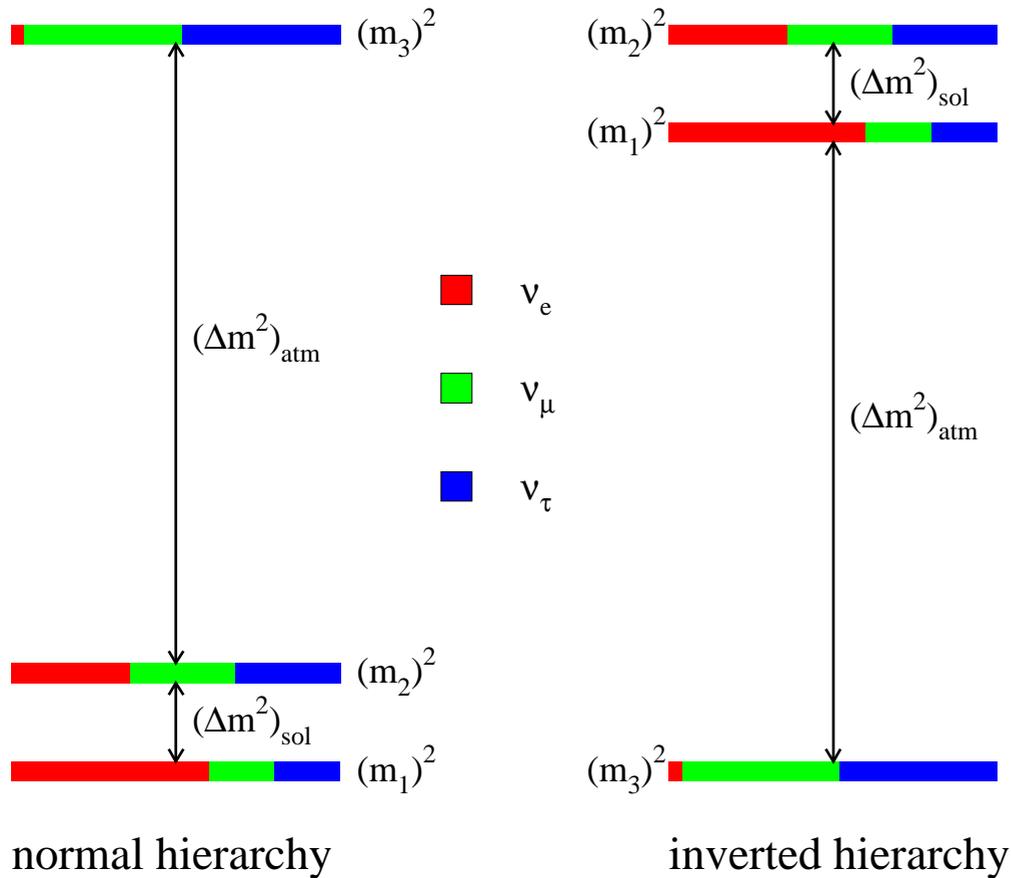
GS98 with Gallium cross-section from [24]	AGSS09 with modified Gallium cross-section [16]
$\Delta m_{21}^2 = 7.59 \pm 0.20 \left(\begin{smallmatrix} +0.61 \\ -0.69 \end{smallmatrix} \right) \times 10^{-5} \text{ eV}^2$	Same
$\Delta m_{31}^2 = \begin{cases} -2.36 \pm 0.11 (\pm 0.37) \times 10^{-3} \text{ eV}^2 \\ +2.46 \pm 0.12 (\pm 0.37) \times 10^{-3} \text{ eV}^2 \end{cases}$	Same
$\theta_{12} = 34.4 \pm 1.0 \left(\begin{smallmatrix} +3.2 \\ -2.9 \end{smallmatrix} \right)^\circ$	$34.5 \pm 1.0 \left(\begin{smallmatrix} +3.2 \\ -2.8 \end{smallmatrix} \right)^\circ$
$\theta_{23} = 42.8 \begin{smallmatrix} +4.7 \\ -2.9 \end{smallmatrix} \left(\begin{smallmatrix} +10.7 \\ -7.3 \end{smallmatrix} \right)^\circ$	Same
$\theta_{13} = 5.6 \begin{smallmatrix} +3.0 \\ -2.7 \end{smallmatrix} (\leq 12.5)^\circ$	$5.1 \begin{smallmatrix} +3.0 \\ -3.3 \end{smallmatrix} (\leq 12.0)^\circ$
$[\sin^2 \theta_{13} = 0.0095 \begin{smallmatrix} +0.013 \\ -0.007 \end{smallmatrix} (\leq 0.047)]$	$[0.008 \begin{smallmatrix} +0.012 \\ -0.007 \end{smallmatrix} (\leq 0.043)]$
$\delta_{\text{CP}} \in [0, 360]$	Same

[Gonzalez-Garcia, Maltoni, Salvado, arXiv:1001.4524]

* Modulo “Anomalies”. Comments Later.

What We Know We Don't Know: Missing Oscillation Parameters

[Driving Force of Next-Generation Oscillation Program]



- What is the ν_e component of ν_3 ? ($\theta_{13} \neq 0$?)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?)
- Is ν_3 mostly ν_μ or ν_τ ? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4$?)
- What is the neutrino mass hierarchy? ($\Delta m_{13}^2 > 0$?)

\Rightarrow All of the above can “only” be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

What we have **really measured** (roughly):

- Two mass-squared differences, at several percent level – many probes;
- $|U_{e2}|^2$ – solar data;
- $|U_{\mu2}|^2 + |U_{\tau2}|^2$ – solar data;
- $|U_{e2}|^2|U_{e1}|^2$ – KamLAND;
- $|U_{\mu3}|^2(1 - |U_{\mu3}|^2)$ – atmospheric data, K2K, MINOS;
- $|U_{e3}|^2(1 - |U_{e3}|^2)$ (upper bound) – reactors (1 km), MINOS, T2K(!?).

We still have a ways to go!

Along the way to the Muon Collider we expect ...

- reactors, T2K, and *No ν a*: $\theta_{13} \neq 0$ (\Rightarrow), hint of mass hierarchy(?)
- LBNE (or equivalent superbeam): mass hierarchy, CP-violation from conventional neutrino beam.
- NuFact(s): high energy and/or low energy.
 - Intense, very well known beam – energy spectrum and flux.
 - high energy ν_e beam!
 - potential for “precision” measurements, potential to significantly over-constrain parameter space(?)

Just for fun ... (I am not advocating this should be taken too seriously yet!)

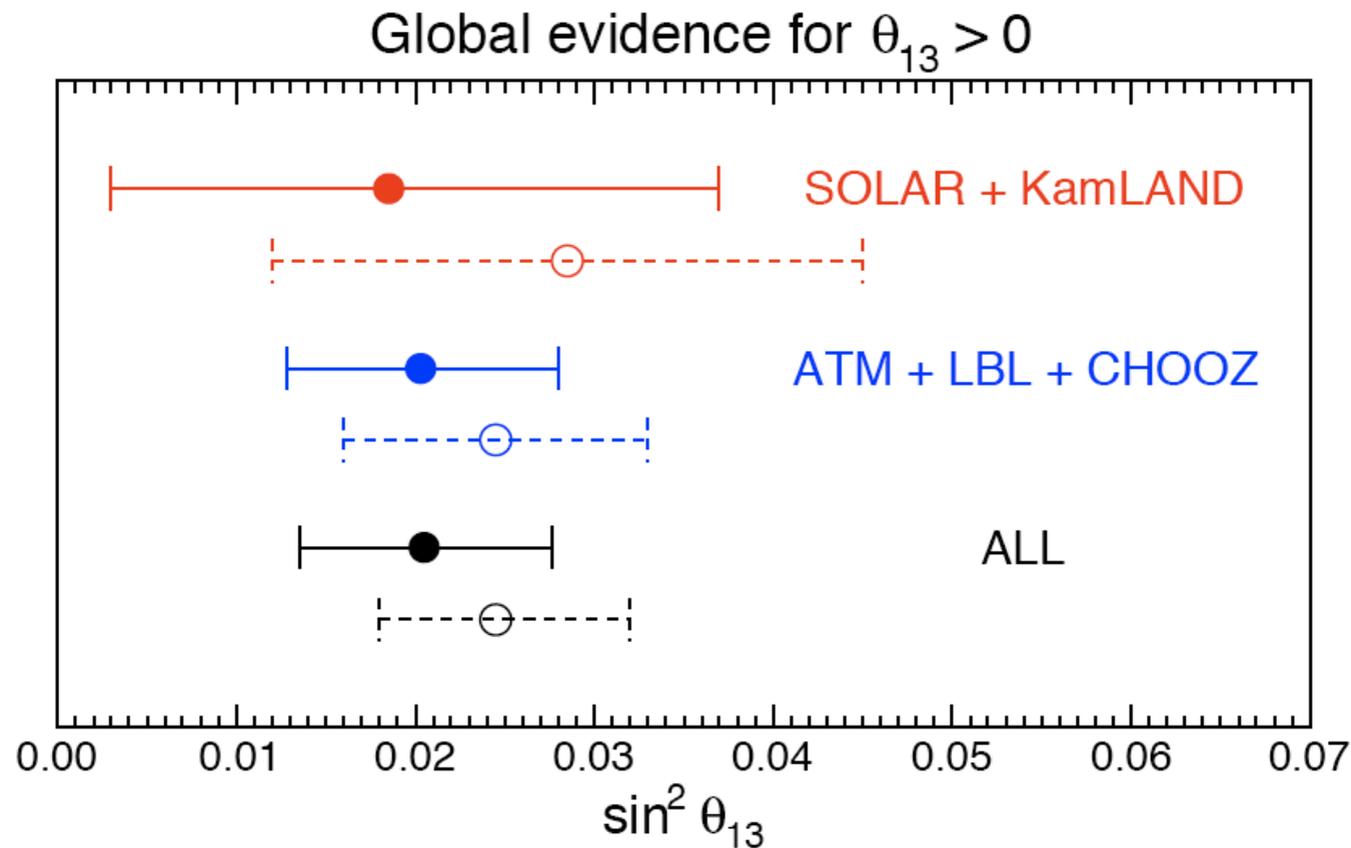
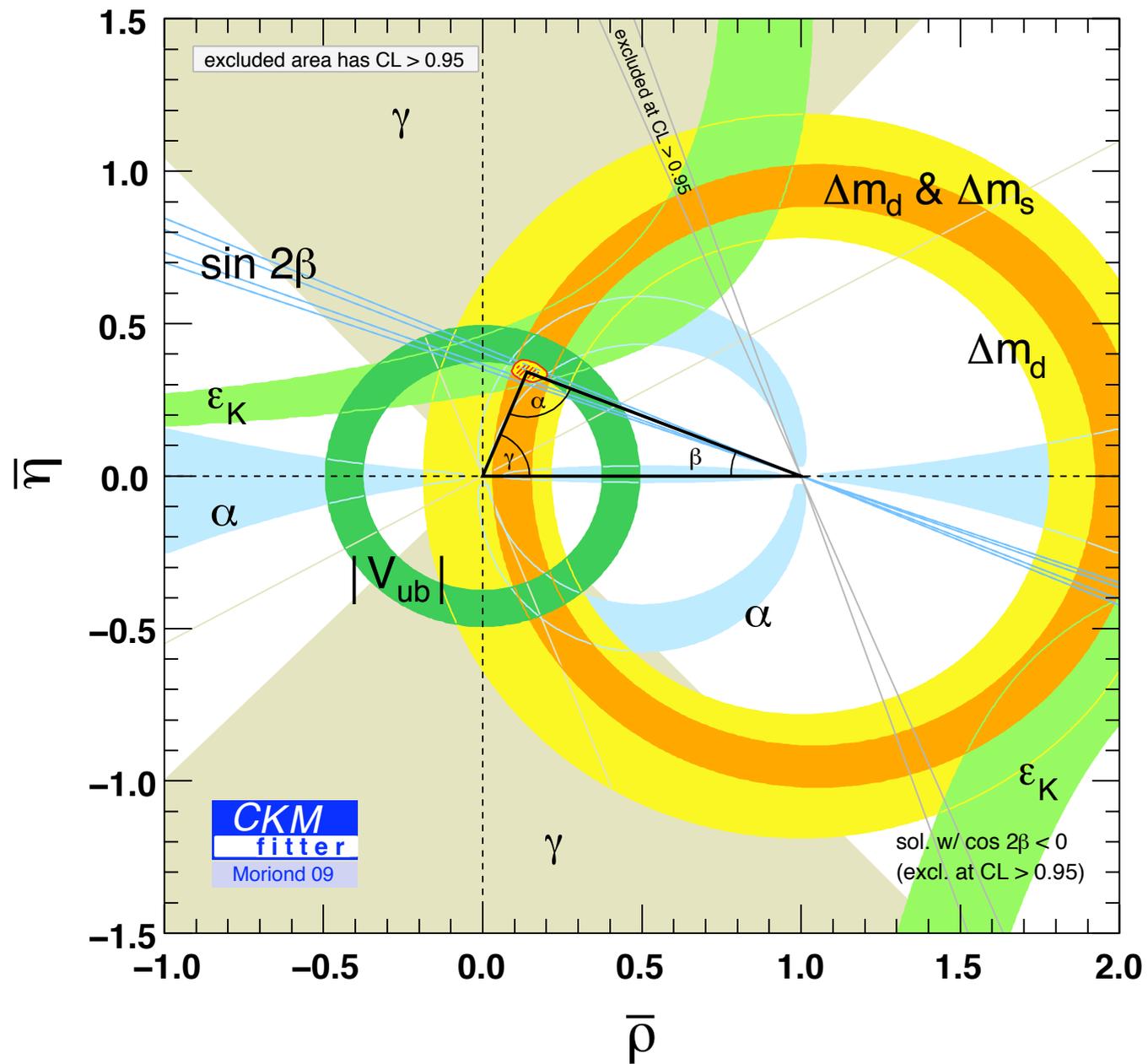


FIG. 3: Global 3ν analysis. Preferred $\pm 1\sigma$ ranges for the mixing parameter $\sin^2 \theta_{13}$ from partial and global data sets. Solid and dashed error bars refer to old and new reactor neutrino fluxes, respectively.

Fogli *et al.*, arXiv:1106.6028.



We need to do this in the lepton sector!

⇒ NEED NuFact.

Evidence(?) For Physics Beyond the Three–Massive–Neutrinos Paradigm

- LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$;
- MiniBooNE $\nu_\mu \rightarrow \nu_e$;
- MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$;
- Reactor Anomaly;
- MINOS ν_μ versus $\bar{\nu}_\mu$ oscillations;
- ...

(Some) Phenomenological Explanations

- Sterile Neutrinos (light, stable variety); LSND, MB, Reactor
- New Neutrino Interactions; MINOS
- Lorentz Invariance/CPT-Violation; “all”
- Sterile Neutrinos (heavy, unstable variety). LSND, MB

Most important: how do we tell which (if any) are correct?

Answer is in the intensity frontier, especially in new short-baseline neutrino experiments.

[new mixing parameters: $|U_{e4,5}|, |U_{\mu4,5}| \sim 0.13 - 0.16$]

	LSND+MB($\bar{\nu}$) vs rest appearance vs disapp.			
	old	new	old	new
$\chi_{\text{PG},3+2}^2/\text{dof}$	25.1/5	19.9/5	19.9/4	14.7/4
PG ₃₊₂	10^{-4}	0.13%	5×10^{-4}	0.53%
$\chi_{\text{PG},1+3+1}^2/\text{dof}$	19.6/5	16.0/5	14.4/4	10.6/4
PG ₁₊₃₊₁	0.14%	0.7%	0.6%	3%

Table III: Compatibility of data sets [23] for 3+2 and 1+3+1 oscillations using old and new reactor fluxes.

data, although in this case the fit is slightly worse than a fit to appearance data only (dashed histograms). Note that MiniBooNE observes an event excess in the lower part of the spectrum. This excess can be explained if only appearance data are considered, but not in the global analysis including disappearance searches [8]. Therefore, we follow [19] and assume an alternative explanation for this excess, e.g. [25]. In Tab. III we show the compatibility of the LSND/MiniBooNE($\bar{\nu}$) signal with the rest of the data, as well as the compatibility of appearance and disappearance searches using the PG test from [23].

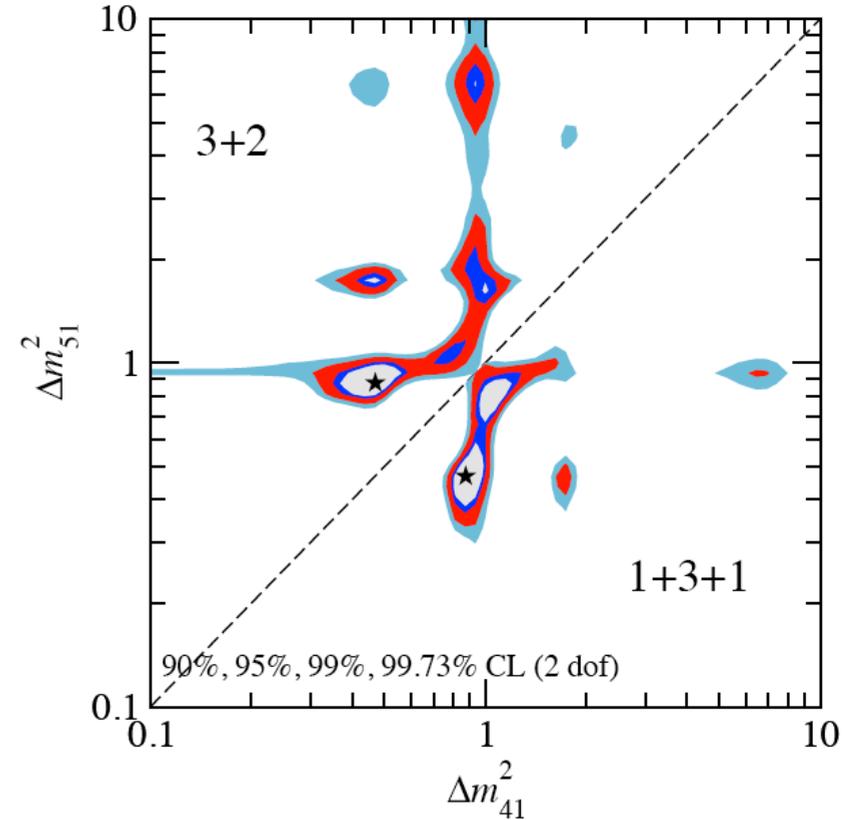


Figure 5: The globally preferred regions for the neutrino mass squared differences Δm_{41}^2 and Δm_{51}^2 in the 3+2 (upper left) and 1+3+1 (lower right) scenarios.

[Kopp, Maltoni, Schwetz, 1103.4570]

Short-Baseline Neutrino Experiments

Short: GeV neutrino energies, baselines of $\sim 100 - 1000$ m. This is “near-detector” physics!

The goals are well-defined and include:

- ν_μ disappearance at the few percent level. Aiming at $4|U_{\mu 4,5}|^2 \sim 0.06$.
- ν_e appearance in the Mini-BooNE LSND range, more accurate. Aiming at $4|U_{e 4,5}|^2|U_{\mu 4,5}|^2 \sim 0.001$.
- Can we see an oscillation? Challenging for these short (only upper bounds for now) baselines...
- How about ν_τ ? If $|U_{\tau 4,5}| \sim 0.1$, expect $P(\nu_\mu \rightarrow \nu_\tau) \sim 0.001$.

Along the way to the Muon Collider we expect ...

- Project X: intense conventional neutrino beams. Dedicated searches for ν_μ disappearance? Need more than just statistics...
- Project X: Great opportunity to build a dedicated $\nu_\mu \rightarrow \nu_\tau$ appearance experiment aiming at $P(\nu_\mu \rightarrow \nu_\tau) \sim 10^{-5}$ or larger (Adam Para).
- Muon storage rings: great for ν_μ disappearance! May be possible to get to 1% with modest effort (Alan Bross).
- Muon storage rings: many more opportunities – ν_e disappearance, $\nu_\mu \rightarrow \nu_e$, etc.

Lots of activity over the past several months. Expect more in the next several months!

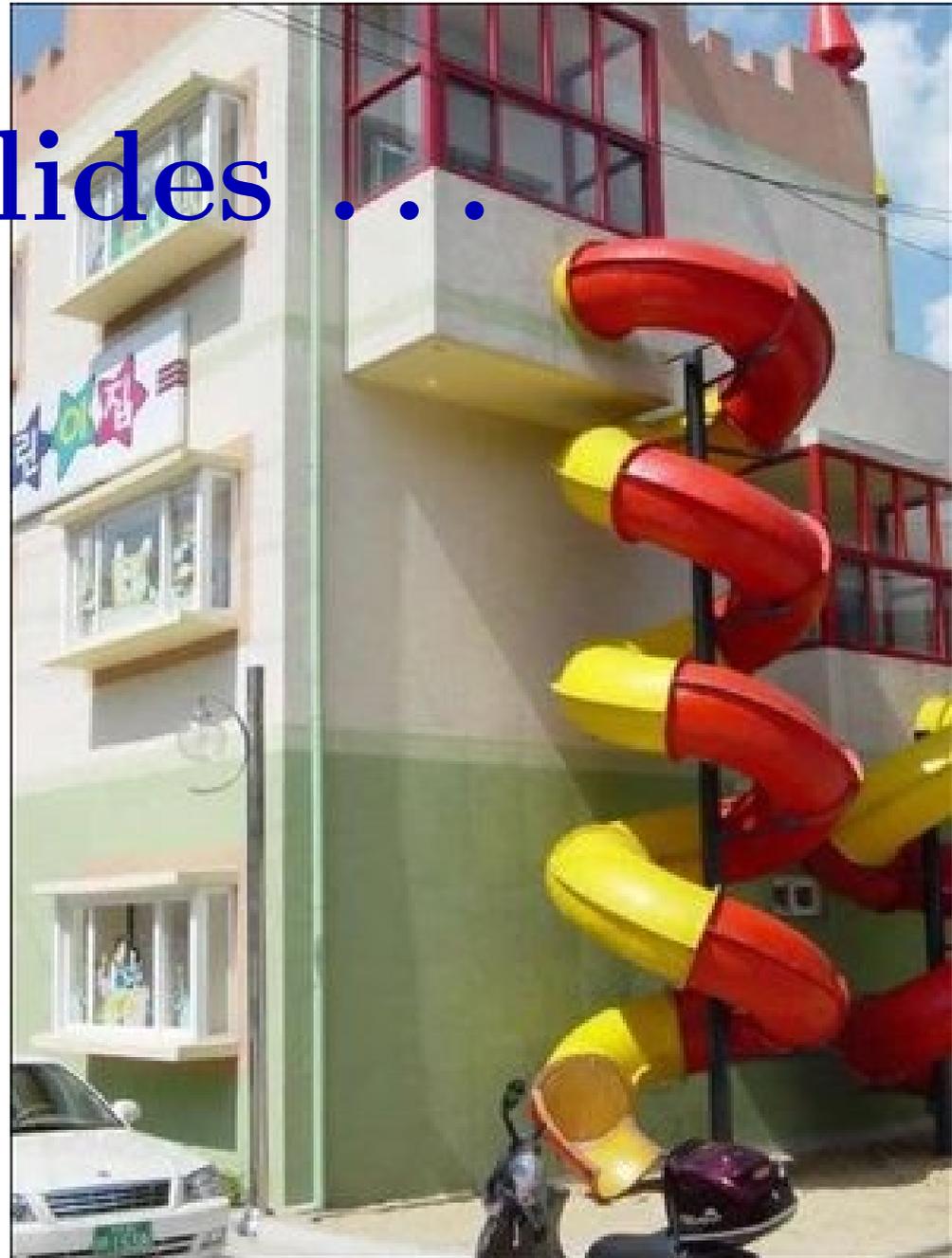
- Short-Baseline Neutrino Workshop (FNAL, May 12–14, 2011)
- <http://www.ft.uam.es/workshops/neutrino/programme.html>

Concluding Thoughts

1. One of the most exciting features of the muon collider program is that at every stage between now and then there is very compelling physics to be pursued.
2. On the flip side, it is imperative, if we are ever to get there, that every stage is justified in its own right. Realistically, this appears to be the only way to get to a multi-TeV muon collider.
3. Most of the physics opportunities lie in the so-called intensity frontier. I discussed some of the physics associated to future muon and neutrino experiments. There are opportunities in kaon physics and other topics (including those outside of particle physics) that I did not discuss...
4. Unlike the muon collider, the physics case for intense proton sources, conventional neutrino beams, muon storage rings, and neutrino factories can be made now, and is independent from future LHC discoveries. Cases can, of course, get stronger!

5. MUONS: very clear path to explore potentially very high scale physics (CLFV). May provide guidance regarding the energy frontier even if none comes from the LHC!
6. NEUTRINOS: very clear path. Potential for game-changing discoveries (new fermions, new weaker-than-weak interactions, etc).
7. Bottom line: The path to the muon collider is filled with priceless physics opportunities. It is possible (plausible?) that, long before we get to collide a muon with an anti-muon, we will have learned something that will ultimately revolutionize the way we describe Nature at very small distance scales!

Backup Slides . . .



Strawman New Physics: New Neutrino–Matter Interactions

These are parameterized by effective four-fermion interactions, of the type:

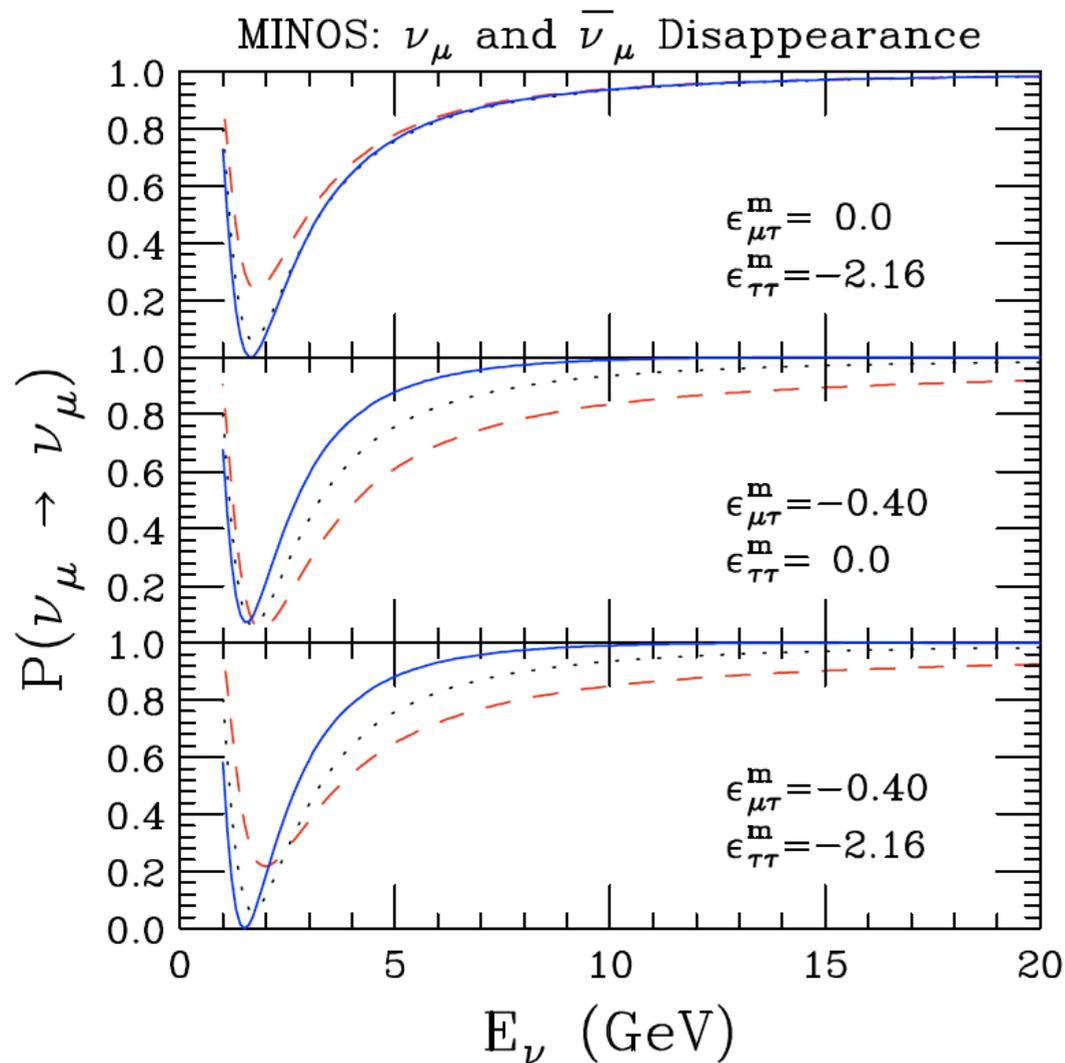
$$L^{NSI} = -2\sqrt{2}G_F (\bar{\nu}_\alpha \gamma_\mu \nu_\beta) \left(\epsilon_{\alpha\beta}^{f\tilde{f}L} \bar{f}_L \gamma^\mu \tilde{f}_L + \epsilon_{\alpha\beta}^{f\tilde{f}R} \bar{f}_R \gamma^\mu \tilde{f}_R \right) + h.c.$$

where $f, \tilde{f} = u, d, \dots$ and $\epsilon_{\alpha\beta}^{f\tilde{f}}$ are dimensionless couplings that measure the strength of the four-fermion interaction relative to the weak interactions.

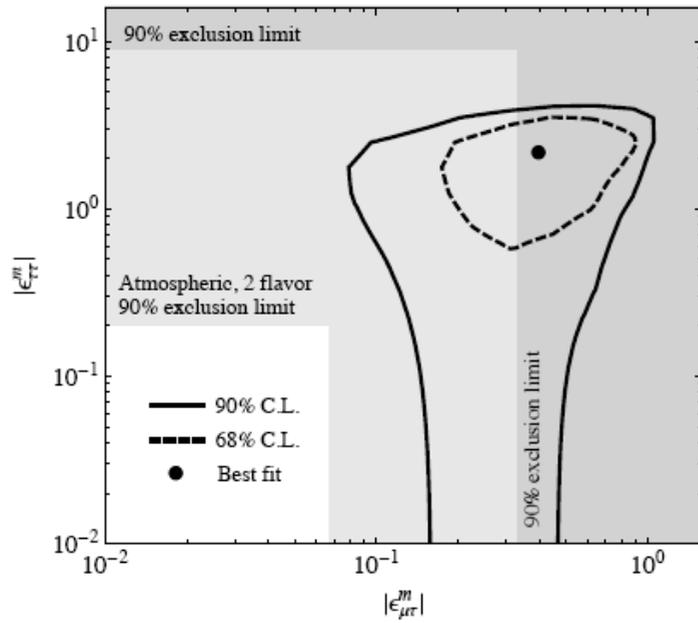
While some of the ϵ s are well constrained (especially those involving muons), some are only very poorly known. These are best searched for in neutrino oscillation experiments, where they mediate **anomalous matter effects**:

$$H_{\text{mat}} = \sqrt{2}G_F n_e \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu}^* & \epsilon_{e\tau}^* \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}^* \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}, \quad \epsilon_{\alpha\beta} = \sum_{f=u,d,e} \epsilon_{\alpha\beta}^{ff} \frac{n_f}{n_e}$$

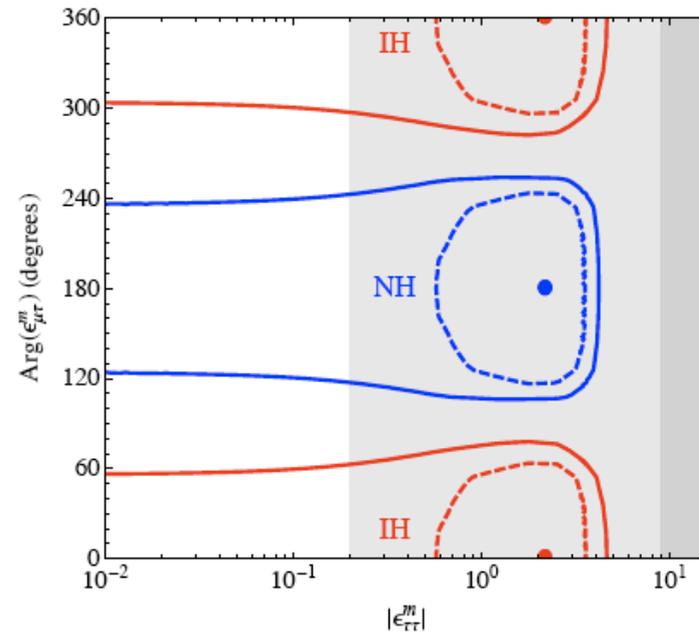
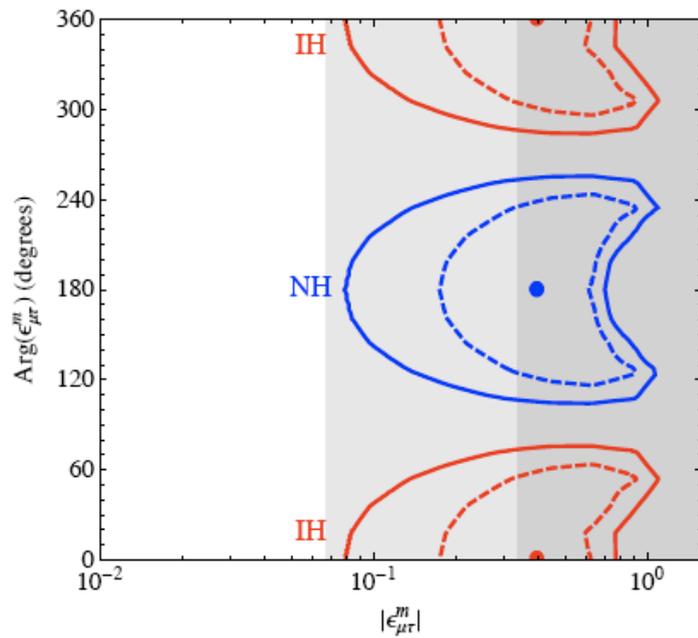
Anomalous matter effects are CPT violating (in a simple, benign way):
neutrinos and antineutrinos behave differently!

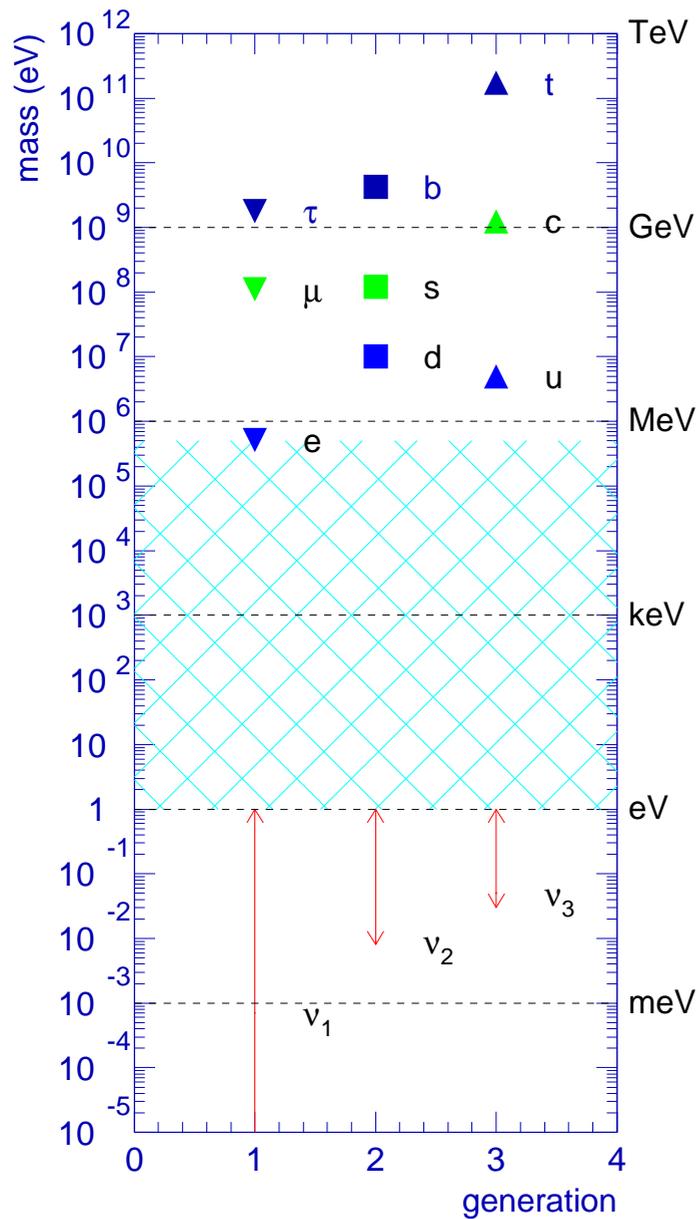


[Kopp, Machado, Parke, 1009.0014]



[Kopp, Machado, Parke, 1009.0014]





What We Are Trying To Understand:

⇐ **NEUTRINOS HAVE TINY MASSES**

⇓ **LEPTON MIXING IS “WEIRD”** ⇓

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

What Does It Mean?

André de Gouvêa
AdG, Jenkins,
0708.1344 [hep-ph]

**Effective
Operator
Approach**

(there are 129
of them if you
discount different
Lorentz structures!)

classified by Babu
and Leung in
NPB619,667(2001)

June 30, 2011

13	$L^i L^j \bar{Q}_i \bar{u}^c L^l e^c \epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda}$	2×10^5	$\beta\beta\nu$
14 _a	$L^i L^j \bar{Q}_k \bar{u}^c Q^k d^c \epsilon_{ij}$	$\frac{y_d y_u g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^3	Northwest $\beta\beta\nu$
14 _b	$L^i L^j \bar{Q}_i \bar{u}^c Q^l d^c \epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda}$	6×10^5	$\beta\beta\nu$
15	$L^i L^j L^k d^c \bar{L}_i \bar{u}^c \epsilon_{jk}$	$\frac{y_d y_u g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^3	$\beta\beta\nu$
16	$L^i L^j e^c d^c \bar{e}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
17	$L^i L^j d^c d^c \bar{d}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
18	$L^i L^j d^c u^c \bar{u}^c \bar{u}^c \epsilon_{ij}$	$\frac{y_d y_u g^4}{(16\pi^2)^4} \frac{v^2}{\Lambda}$	2	$\beta\beta\nu$, LHC
19	$L^i Q^j d^c d^c \bar{e}^c \bar{u}^c \epsilon_{ij}$	$y_\ell y_\beta \frac{y_d^2 y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1	$\beta\beta\nu$, HEInν, LHC, m
20	$L^i d^c \bar{Q}_i \bar{u}^c \bar{e}^c \bar{u}^c$	$y_\ell y_\beta \frac{y_d y_u^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	40	$\beta\beta\nu$, mix
21 _a	$L^i L^j L^k e^c Q^l u^c H^m H^n \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	$\beta\beta\nu$
21 _b	$L^i L^j L^k e^c Q^l u^c H^m H^n \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	$\beta\beta\nu$
22	$L^i L^j L^k e^c \bar{L}_k \bar{e}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
23	$L^i L^j L^k e^c \bar{Q}_k \bar{d}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	40	$\beta\beta\nu$
24 _a	$L^i L^j Q^k d^c Q^l d^c H^m \bar{H}_i \epsilon_{jk} \epsilon_{lm}$	$\frac{y_d^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^2	$\beta\beta\nu$
24 _b	$L^i L^j Q^k d^c Q^l d^c H^m \bar{H}_i \epsilon_{jm} \epsilon_{kl}$	$\frac{y_d^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	1×10^2	$\beta\beta\nu$
25	$L^i L^j Q^k d^c Q^l u^c H^m H^n \epsilon_{im} \epsilon_{jn} \epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	4×10^3	$\beta\beta\nu$
26 _a	$L^i L^j Q^k d^c \bar{L}_i \bar{e}^c H^l H^m \epsilon_{jl} \epsilon_{km}$	$\frac{y_\ell y_d}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	40	$\beta\beta\nu$
26 _b	$L^i L^j Q^k d^c \bar{L}_k \bar{e}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_\ell y_d}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	40	$\beta\beta\nu$
27 _a	$L^i L^j Q^k d^c \bar{Q}_i \bar{d}^c H^l H^m \epsilon_{jl} \epsilon_{km}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
27 _b	$L^i L^j Q^k d^c \bar{Q}_k \bar{d}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
28 _a	$L^i L^j Q^k d^c \bar{Q}_j \bar{u}^c H^l \bar{H}_i \epsilon_{kl}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
28 _b	$L^i L^j Q^k d^c \bar{Q}_k \bar{u}^c H^l \bar{H}_i \epsilon_{jl}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
28 _c	$L^i L^j Q^k d^c \bar{Q}_i \bar{u}^c H^l \bar{H}_i \epsilon_{jk}$	$\frac{y_d y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^3	$\beta\beta\nu$
29 _a	$L^i L^j Q^k u^c \bar{Q}_k \bar{u}^c H^l H^m \epsilon_{il} \epsilon_{jm}$	$\frac{y_u^2}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^5	$\beta\beta\nu$
29 _b	$L^i L^j Q^k u^c \bar{Q}_i \bar{u}^c H^l H^m \epsilon_{ik} \epsilon_{jm}$	$\frac{g^2}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	4×10^4	$\beta\beta\nu$
30 _a	$L^i L^j \bar{L}_i \bar{e}^c \bar{Q}_k \bar{u}^c H^k H^l \epsilon_{jl}$	$\frac{y_\ell y_u}{(16\pi^2)^3} \frac{v^2}{\Lambda}$	2×10^3	$\beta\beta\nu$
30 _b	$L^i L^j \bar{L}_m e^c \bar{Q}_n u^c H^k H^l \epsilon_{ik} \epsilon_{jl} \epsilon^{mn}$	$\frac{y_\ell y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	2×10^3	Intense Physics $\beta\beta\nu$
31 _a	$L^i L^j \bar{Q}_i \bar{d}^c \bar{Q}_i \bar{u}^c H^k H^l \epsilon_{ij}$	$\frac{y_d y_u}{(16\pi^2)^2} \frac{v^2}{\Lambda} \left(\frac{1}{16\pi^2} + \frac{v^2}{\Lambda^2} \right)$	4×10^3	$\beta\beta\nu$

